Cosmology from M Theory Compactifications with Fluxes: Cosmic Speed-Up and Dark Energy

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- Motivations
- Accelerated Universes: Possibilities
- Compactification on hyperbolic spaces
- Flat universes and transient acceleration
- Open universes and eternal acceleration
- Dark energy, modified gravity, and related issues

✧ Based on

- **hep-th/0304177** (JHEP:0307:017)
(with Pei-Ming Ho, Chiang-Mei Chen and John Wang)
- **hep-th/0306291** (JHEP:0310:058)
(with Pei-Ming Ho, Chiang-Mei Chen, Nobuyoshi Ohta and John Wang)
- **hep-th/0309139** (NPB, Supplement)
- **hep-th/0311071**

String or M Theory Cosmology: Motivations

Recent interest in string or M-theory cosmology is two fold

(1) Can one derive a scalar potential from string/M theory that has stationary points (or positive extremum) with $V > 0$.

(2) Can inflation and/or late-time acceleration of the four dimensional expanding universe (naturally) emerge from M-theory compactifications

The issue whether compactifications of d=10 or d=11 string or M-theory down to d=4 should be time-independent or time-dependent may be discussed. If the first approach is closer to reality then "dark energy" is possibly a pure vacuum energy, so called cosmological constant. While if the compactification can be time-dependent, then the dark energy is dynamical, for example, a slowly varying scalar field, which may arise due to a slowly varying size of extra dimensions.

Many tentative effective models may be devised to explain primordial inflation as well as late-time acceleration of the universe

But the central question is

Is 4d cosmology derivable from string/M theory compactification?

No-go theorem in a language of cosmology?

One of the obstacles for a de Sitter type compactification in supergravity theories is no-go theorem Gibbons (1984), Maldacena-Nunez (2001)

The strong energy condition $\mathbf{R}_{00}^{(D)} \geq 0$ holds for 10 or 11d supergravities. If one allows extra dimensions to be warped and time-independent, then for a compactified theory this requires

$$\mathbf{R}_{00}^{(4)} \geq 0$$

But this does not allow the universe to accelerate! So it must be violated during inflation. Consider a four-dimensional metric

$$\mathbf{ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega_2^2 \right)},$$

where $k = -1,\, 0,\, +1$. The time-time component of 4D Ricci tensor is

$$\mathbf{R}_{00}^{(4)} = -\,\mathbf{3}\,\frac{\ddot{\mathbf{a}}(\mathbf{t})}{\mathbf{a}(\mathbf{t})}$$

Inflating spacetime means $\ddot{a}/a > 0$ and hence $R_{00}^{(4)} < 0$

No-go theorem in warped string compactifications

How the Gibbons-Maldacena-Nunez "no-go theorem" comes into this business? Consider a D-dimensional metric

$$\mathbf{ds_D^2 = A^2(y)\, ds_4^2(x) + d\Sigma_m^2(y) \quad (m = D - 4)}$$

$d\Sigma_m^2$ is the metric of some compact non-singular m-manifold $\mathcal{M}$ with coordinates y. One computes

$$\mathbf{R_{00}^{(D)}(x,y) = R_{00}^{(4)}(x) - \frac{1}{4}\, A^{-2}(y)\, \nabla_y^2\, A^4(y)}$$

Multiplying by A^2 and integrating over $\mathcal{M}$ we find

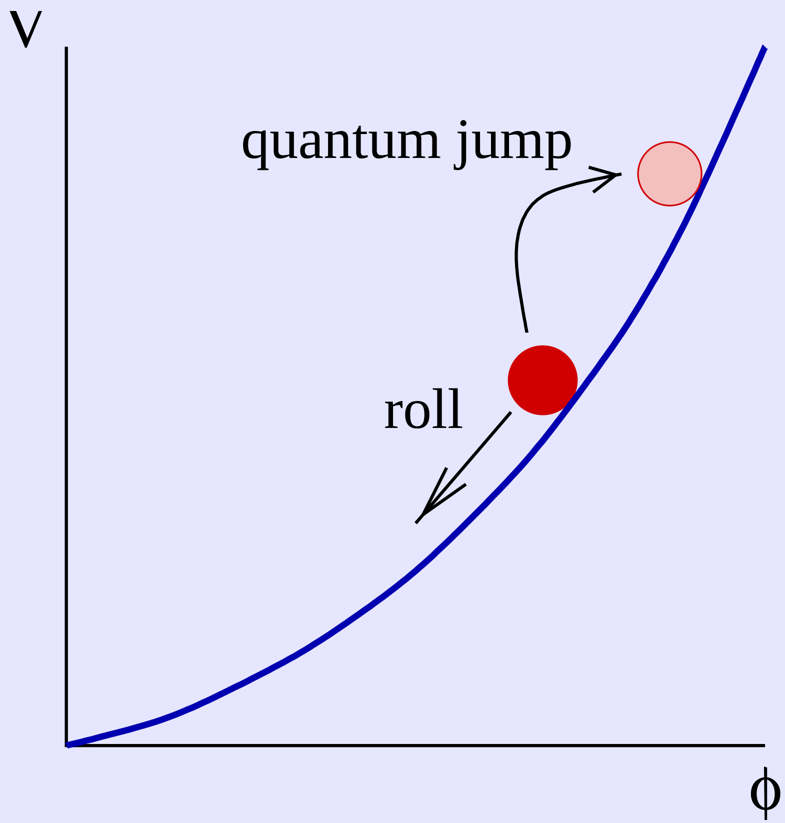
$$\left[\int_{\mathcal{M}} \mathbf{A^2}\right] \mathbf{R_{00}^{(4)}} = \int_{\mathcal{M}} \mathbf{A^2\, R_{00}^{(D)}}$$

The result is that

$$\mathbf{R_{00}^{(D)} \geq 0} \quad \text{only} \quad \text{if} \quad \mathbf{R_{00}^{(4)} \geq 0}$$

A contribution of p-form background fields does not affect this result!

What really violated during inflation?



Roll $\Longrightarrow$ lower $\mathbf{V}$ $\Longrightarrow$ lower $\mathbf{H}$ (Hubble parameter)

Jump $\Longrightarrow$ higher $\mathbf{V}$ $\Longrightarrow$ higher $\mathbf{H}$

What conditions really violated during inflation?

FRW:

$$\mathbf{\dot{H} = -4\pi G(\rho + p) + \frac{k}{a^2}}$$

Inflating spacetime does not mean that $|k| = 0$, rather

$$\mathbf{\frac{k}{a^2} \to 0} \quad \text{or} \quad \text{negligibly} \quad \text{small}$$

The condition $\dot{H} > 0$ is too strong! This implies that

$$\mathbf{\frac{\ddot{a}}{a} > H^2 > 0 \Rightarrow \rho + p < 0}$$

$$\mathbf{\Rightarrow N^\mu N^\nu T_{\mu\nu} < 0 \Rightarrow w = \frac{p}{\rho} < -1}$$

Violation of NEC $\Leftrightarrow$ Phantom Fields? Probably NOT!

In general, cosmic acceleration means that

$$\mathbf{\frac{\ddot{a}}{a} > 0, \quad H > 0 \Rightarrow -1 < w < -\frac{1}{3}}$$

Resolution: Hyperbolic Compactification

How To Overcome No-Go Theorem?

• It is possible to explain cosmic acceleration of our four-dimensional universe from supergravity solutions, with or without background fluxes,

if

1) one gives up the condition of time-independence of internal space, and in addition,

2) the internal space is hyperbolic (a space of constant negative curvature)

——- Townsend and Wolhfarth [hep-th/0303097]

This is an interesting observation

Similar cosmology was studied by E. Kasner in 1921

Look! This is right after T. Kaluza but before O. Klein

Kasner solutions

The Kasner-type metric

$$\mathbf{ds^2 = -dt^2 + a(t)^2\, dx^2_{\mathbb{R}_3} + b(t)^2\, ds^2(T^m)}$$

solves the $4+m$ dimensional vacuum Einstein equations if

$$\mathbf{a(t) = t^{\alpha}}\,, \quad \mathbf{b(t) = t^{\beta}}$$

$$\alpha = \frac{\mathbf{3 \pm \sqrt{3m(m+2)}}}{\mathbf{3(m+3)}}\,, \quad \beta = \frac{\mathbf{m \mp \sqrt{3m(m+2)}}}{\mathbf{m(m+3)}}$$

Of course $\alpha < 1$ and so these solutions in general do not give accelerating expansion of 4d spacetime. However, by using the time-shift symmetry

$$t \to t_{\infty} - t\,, \quad a(t) = (t_{\infty} - t)^{\alpha}$$

one can see that the scale factor yields accelerated expansion since $\dot{a} > 0$ and $\ddot{a} > 0$. Unfortunately, in Einstein conformal-frame we find that the solution is actually decelerating!

Accelerating Cosmologies: Some Known Examples

• $\mathbb{M}_4 \times \mathbb{H}_m$: – **First example of transient acceleration of a four-dimensional universe from supergravity compactification on hyperbolic spaces –Townsend-Wohlfarth (hep-th/0303097)**

• **The solution is**

$$ds^2 = e^{-m\phi(t)} \left(-S^6\, dt^2 + S^2\, dx_3^2\right) + r_c^2\, e^{2\phi(t)}\, ds^2_{H_m}$$

$$\phi(t) = \frac{1}{m-1}\Big(\ln K(t) - 3\lambda_0 t\Big), \quad S^2 = K^{\frac{m}{m-1}}\, e^{-\frac{(m+2)}{(m-1)}\lambda_0 t}$$

$$K(t) = \frac{\lambda_0 r_c}{(m-1)} \frac{\beta}{\sinh\left[\lambda_0 \beta |t+t_1|\right]}, \quad \beta = \sqrt{3 + \frac{6}{m}}$$

• **This solution is obtainable from Space-like brane solutions in zero-flux limit – Ohta [hep-th/0303238] .**

• **The proper time τ is defined by $d\tau = S^3(t)\, dt$. The conditions for expansion and acceleration are $\frac{dS}{d\tau} > 0$, $\frac{d^2S}{d\tau^2} > 0$. For example, when $m = 7$, the expansion factor is simply**

$$\frac{S(\tau_2)}{S(\tau_1)} = 3.04 \quad \text{Too small for inflation}$$

Other Accelerating Solutions

• $\mathbb{M}_4 \times \mathbb{R}_{m_1} \times \mathbb{H}_{m_2}$: **Suppose we live in a flat $4D$ spacetime, and the internal space is a product of flat and hyperbolic spaces.**

The logarithm of the scale factor is

$$\ln(\mathbf{S}(\mathbf{t})) = -\frac{(\mathbf{m_1}+\mathbf{m_2}-\mathbf{4})}{\mathbf{4}}\lambda_\mathbf{0}\mathbf{t} + \frac{\mathbf{m_1}}{\mathbf{2}}\mathbf{a}(\mathbf{t}) + \frac{\mathbf{m_2}}{\mathbf{2}}\mathbf{b}(\mathbf{t})$$

$$\mathbf{a}(\mathbf{t}) = \alpha_\mathbf{0}\mathbf{t}\,, \quad \mathbf{b}(\mathbf{t}) = -\frac{\mathbf{m_1}}{\mathbf{m_2}-\mathbf{1}}\,\alpha_\mathbf{0}\mathbf{t} + \frac{\mathbf{1}}{\mathbf{m_2}-\mathbf{1}}\ln\left(\frac{\beta}{\sinh((\mathbf{m_2}-\mathbf{1})\beta\,\mathbf{t})}\right)$$

For example, when $m_1 = 1$ and $m_2 = 6$ and $\lambda_0 = 2\alpha_0$, so the volume factor of $\mathbb{R}^1$ is unity, then the four-dimensional universe accelerates in the time interval $\mathbf{1.41} > \mathbf{4}\alpha_\mathbf{0}\mathbf{t} > \mathbf{0.14}$. The expansion factor is

$$\frac{\mathbf{S}(\tau_\mathbf{2})}{\mathbf{S}(\tau_\mathbf{1})} = \mathbf{3.38} \quad \text{Just a small improvement}$$

Hyperbolic-Flux Compactification

• The Kasner type solutions were first studied and generalized in supergravity by A. Chodos and S. Detweiler in 1980 by considering the bosonic part of D-dimensional supergravity with a $(q+2)$-form field:

$$\mathcal{L}_D = \frac{1}{16\pi G_D}\sqrt{-g_D}\left(R - \frac{8\pi G_D}{(q+2)!}F^2_{[q+2]}\right)$$

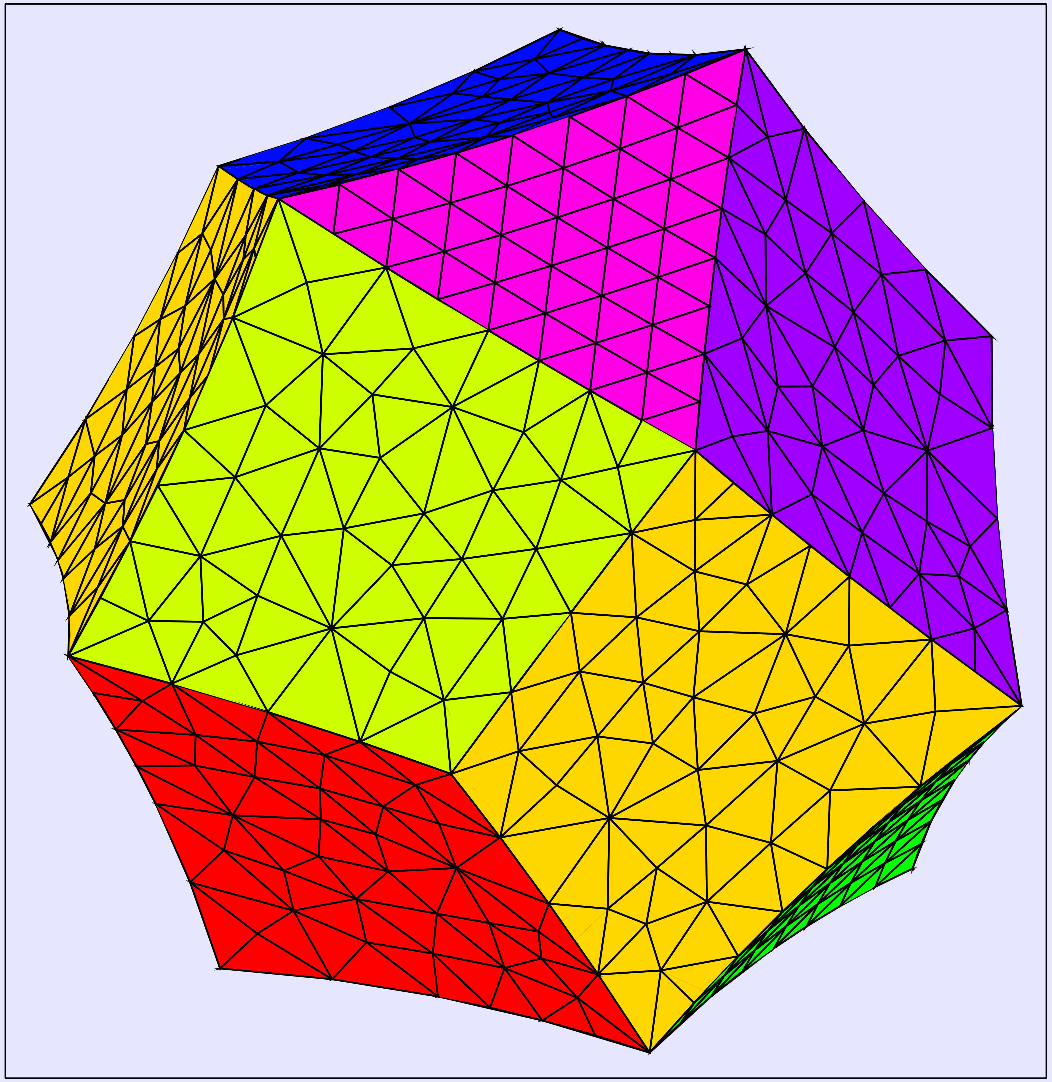
where $F_{(q+2)} = dA_{(q+1)}$. The metric in Einstein conformal-frame reads

$$ds_D^2 = e^{-\frac{2m}{d-2}\phi(t)}\left[-dt^2 + a(t)^2\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega_{d-2}^2\right)\right] + r_c^2\, e^{2\phi(t)} d\Sigma^2_{m,k_1}$$

and $d = q+2$. The values of $k_i = -1,\, 0,\, +1$ correspond to the hyperbolic, flat or spherical space. In $D = 11$ (i.e., $d = 4$, $m = 7$), one has 4-form anti-symmetric tensor matter fields as required by supersymmetry.

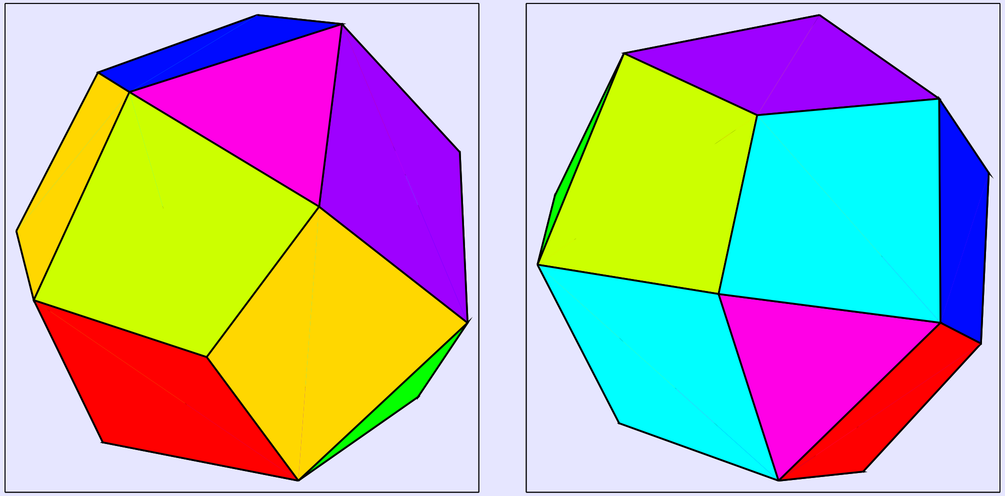
Chodos and Detweiler studied the $k_1 = 0$ cosmology, presumably, due to a mathematical simplicity. Besides, the metric was not written in Einstein conformal-frame, leading to the result that $G_4 \propto t^{-1}$, unacceptable!

Compact Hyperbolic Manifold



Compact Hyperbolic 3-Space – Thurston Manifold

Compact Hyperbolic Manifold



Compact Hyperbolic 3-Space – In Klein Coordinates (using SnapPea)

SnapPea is a computer program developed in order to study compact hyperbolic spaces, as well as compact hyperbolic orbifolds.

SnapPea can actually compute the volume, fundamental group, symmetry group, homology, Chern-Simon invariant and length spectrum of such spaces

Dimensional Reduction

Upon the dimensional reduction, the d-dimensional Lagrangian density is (IPN – hep-th/0309139)

$$\mathcal{L}_{\mathbf{d}} = \mathbf{M_d^2} \sqrt{-\mathbf{g_d}} \left(\frac{\mathcal{R}}{\mathbf{2}} - \mathbf{\Lambda_d} + \mathbf{K} - \mathbf{V}(\phi) \right)$$

where the kinetic and potential terms are

$$\mathbf{K} = -\frac{\lambda}{\mathbf{2}} \mathbf{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

$$\frac{\mathcal{R}}{2} - \mathbf{\Lambda_d} = \mathbf{k} \frac{(\mathbf{d}-\mathbf{2})(\mathbf{d}-\mathbf{3})}{\mathbf{2a^2}} + \frac{(\mathbf{d}-\mathbf{1})(\mathbf{d}-\mathbf{2})}{\mathbf{2}} \mathbf{H^2} + (\mathbf{d}-\mathbf{1}) \frac{\ddot{\mathbf{a}}}{\mathbf{a}}$$

$$\mathbf{V}(\phi) = \mathbf{b^2} \mathbf{e}^{-\frac{\mathbf{2(d-1)m}}{\mathbf{d-2}}\phi} - \mathbf{k_1} \frac{\mathbf{m}(\mathbf{m}-\mathbf{1})}{\mathbf{2r_c^2}} \mathrm{e}^{-\frac{\mathbf{2}\lambda}{\mathbf{m}}\phi(\mathbf{t})}$$

with $\lambda \equiv \frac{m(m+d-2)}{(d-2)}$. It turns out that a negatively curved geometry of the internal space (i.e., $k_1 = -1$) gives a positive exponential potential $V(\phi)$ in d-dimensions, even if $b = 0$. In cosmology, practically, one sets $d = 4$.

We should note, on the way to proceed further, that dimensional reduction of any higher-dimensional theory is well defined only if $d \geq 4$, when $k \neq 0$

Can 1/R type gravity be derived from M-theory?

Is gravity of the type $R + 1/R^n$, with $n > 0$, derivable from M-theory? What seems plausible is that one can write an equivalent action containing the term $1/R^n$ with $n < 1$, but it is unlikely that $n \geq 1$ is obtained from classical compactifications of supergravity theories. This holds even if one replaces field strengths by a higher-dimensional CC Λ, so that

$$\mathbf{V}(\phi) = \mathbf{\Lambda}\, \mathbf{e}^{-\frac{2m}{d-2}\phi} - \mathbf{k_1} \frac{\mathbf{m(m-1)}}{\mathbf{2r_c^2}} \mathrm{e}^{-\frac{2\lambda}{m}\phi(t)}$$

Then also one could ask why Carroll et.al.'s model

$$\mathcal{L}[\mathbf{g}_{\mu\nu}] = \frac{\mathbf{M_P^2}}{\mathbf{2}} \sqrt{\mathbf{g}} \left(\mathbf{R} - \frac{\mu^4}{\mathbf{R}} \right)$$

or equivalently

$$\mathcal{L} = \sqrt{-\tilde{\mathbf{g}}} \left(\frac{\mathbf{M_P^2}}{\mathbf{2}} \tilde{\mathbf{R}} - (\partial\sigma)^2 - 2\mu^2 \mathbf{M_P^2}\, \mathrm{e}^{-2\sqrt{\frac{2}{3}}\frac{\sigma}{\mathbf{M_P}}} \sqrt{\mathrm{e}^{\sqrt{\frac{2}{3}}\frac{\sigma}{\mathbf{M_P}}} - 1} \right)$$

gives a power-law inflation? A possible reason is that the asymptotic form of potential is an exponential $V(\sigma) \propto e^{-2\lambda(\sigma/M_p)}$ with $\lambda < 1$. Such an exponential potential is known to give an eternally accelerating expansion, although we do not know yet how to derive an exponential potential with $\lambda < 1$ from string/M-theory compactifications.

Four Dimensional Scalar Potential

- Is it possible to use M-theory motivated potentials for dark energy?

$$\mathcal{L} = \sqrt{-\mathbf{g}}\left[\frac{\mathbf{M_P^2}}{\mathbf{2}}\mathbf{R} - (\partial\varphi)^{\mathbf{2}} - \mathbf{2V}(\varphi)\right]$$

- The $4d$ scalar potential arising from hyperbolic-flux compactification is

$$\mathbf{V}(\varphi) = \frac{\mathbf{M_P^2}}{\mathbf{r_c^2}}\,\mathrm{e}^{-\mathbf{2\,c}\,\frac{\varphi}{\mathbf{M_P}}} + \mathbf{M_P^2}\,\frac{\mathbf{f^2}}{\mathbf{2}}\,\mathbf{e}^{-\,\frac{\mathbf{6}}{\mathbf{c}}\,\frac{\varphi}{\mathbf{M_P}}}$$

PS: If one prefers a normalization of φ such that the kinetic part is $\dot{\varphi}^2/2$ and potential is $V(\varphi)$, then this is attained by the substitution $\varphi \to \varphi/\sqrt{2}$ and $c \to c/\sqrt{2}$

The canonical scalar φ is related to the original ϕ, see the metric ansatz in slide (12), by the relation

$$\phi = \sqrt{\frac{\mathbf{4}}{\mathbf{m(m+2)}}}\,\frac{\varphi}{\mathbf{M_P}} + \frac{\mathbf{1}}{\mathbf{m+2}}\ln\frac{\mathbf{m(m-1)}}{\mathbf{4}}$$

$$\mathbf{f^2} = \mathbf{b^2}\left(\frac{\mathbf{4}}{\mathbf{m(m-1)}}\right)^{\mathbf{3/c^2}}, \quad \mathbf{c} \equiv \sqrt{\frac{\mathbf{m+2}}{\mathbf{m}}}$$

Potential = Cosmological Constant ?

For all classical (string or Kaluza-Klein) compactifications, only $c > 1$ arises in practice. In particular, for the hyperbolic compactification, since $c = \sqrt{\frac{m+2}{m}}$, one has $1 \lesssim c < \sqrt{3}$ when $m \geq 2$.

In the M theory case $m = 7$, and so $c = 3/\sqrt{7}$, we find

$$\mathbf{V}(\varphi) = \frac{\mathbf{M_P^2}}{\mathbf{r_c^2}}\, \mathbf{e}^{-2\sqrt{9/7}\,\frac{\varphi}{\mathbf{M_P}}} + \frac{\mathbf{M_P^2\, f^2}}{\mathbf{2}}\, \mathbf{e}^{-2\sqrt{7}\,\frac{\varphi}{\mathbf{M_P}}}$$

- The late-time cosmology, $\varphi >> 1$, is almost unaffected by the flux term. The first exponent $\sqrt{9/7} \approx 1.1338$ is within the limit where astronomical data might be relevant, $\lambda \lesssim \sqrt{6}/2 = 1.2247$. The extra dimensions $m = 4$ is marginal for which $\lambda = \lambda_{crit}$.

The value of r_c is not fixed by field equations. A reasonable value of r_c is $\sim 1\ TeV \sim 10^{-15}$ cm or even less, but bigger than string scale.

Then the above potential may be tuned to the present value of the cosmological constant, that is, $V(\varphi) \sim 10^{-120}$, in 4d Planck units, given that $\varphi \sim 91$ and $r_c \sim 1$ TeV.

Four Dimensional Cosmology

For the single hyperbolic space as an internal manifold, we find that the φ equation of motion is (for a while, let me work in the units $M^2 = (8\pi G)^{-1} = 1$)

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{2k_1}{r_c^2} c\, e^{-2c\,\varphi} - \frac{3}{c} f^2 e^{-\frac{6}{c}\varphi} = 0$$

while the Friedman equation is

$$3H^2 = \dot{\varphi}^2 - \frac{2k_1}{r_c^2} e^{-2c\,\varphi} - \frac{3k}{a^2} + f^2 e^{-(6/c)\varphi}$$

Note that there is another equation which is independent of the background flux and the (radion/inflaton) coupling c

$$H^2 - \frac{\ddot{a}}{a} + \frac{k}{a^2} = \dot{\varphi}^2$$

• Consider that the four dimensional universe is spatially flat ($k = 0$). In this case it is convenient to define a new logarithmic time variable τ by $d\tau = e^{-c\,\varphi}\, dt\,, \quad \alpha(\tau) = \ln(a(t)) \;\Rightarrow\; a(t) = e^{\alpha(\tau)}$

Examples of Transient Acceleration

In the case of zero flux ($f = 0$) the solution is

$$\sqrt{\mathbf{3}}\,\alpha = \delta_-\,\mathbf{ln}\cosh\frac{\gamma(\tau+\tau_{\mathbf{1}})}{\mathbf{r_c}} + \delta_+\,\ln\sinh\frac{\gamma(\tau+\tau_{\mathbf{1}})}{\mathbf{r_c}} + \mathbf{A}$$

$$\varphi = \delta_-\,\mathbf{ln}\cosh\frac{\gamma(\tau+\tau_{\mathbf{1}})}{\mathbf{r_c}} - \delta_+\,\ln\sinh\frac{\gamma(\tau+\tau_{\mathbf{1}})}{\mathbf{r_c}} + \mathbf{B}$$

$$\delta_\pm = \frac{\mathbf{1}}{\sqrt{\mathbf{3}}\pm\mathbf{c}}, \quad \gamma = \sqrt{\frac{\mathbf{3}-\mathbf{c^2}}{\mathbf{2}}}$$

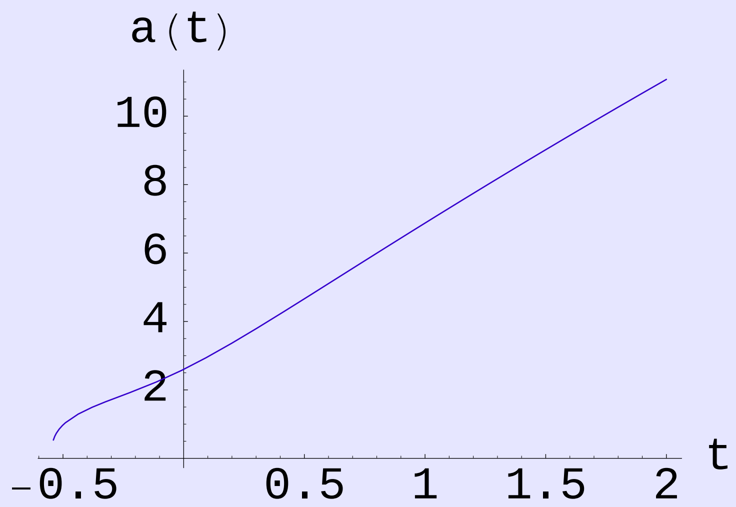
This gives (for simplicity we set $\tau_1 = 0$)

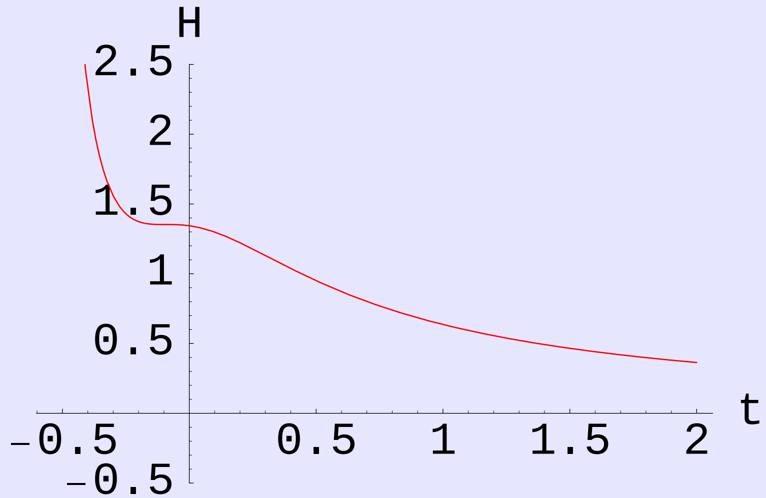
$$\mathbf{H} = \frac{\mathbf{da/dt}}{\mathbf{a}} = \mathrm{e}^{-\mathbf{c}\varphi}\,\alpha'(\tau) > \mathbf{0}$$

$$\frac{\ddot{\mathbf{a}}}{\mathbf{a}} = \mathrm{e}^{-\mathbf{2c}\,\varphi}\frac{\mathbf{2}\gamma^{\mathbf{2}}}{\mathbf{r_c^2}}\left[\frac{\mathbf{2(c^2-1)}}{\mathbf{c^2-3}} + \frac{\mathbf{2}\sqrt{\mathbf{3}}\,\mathbf{c}\left(\mathbf{2}\cosh^{\mathbf{2}}(\gamma\tau/\mathbf{r_c})-\mathbf{1}\right)-\mathbf{c^2}-\mathbf{3}}{\mathbf{3(3-c^2)}\left(\cosh^{\mathbf{2}}(\gamma\tau/\mathbf{r_c})-\mathbf{1}\right)\cosh^{\mathbf{2}}(\gamma\tau/\mathbf{r_c})}\right]$$

The critical value $c = 1$ separates qualitatively the different cosmologies. Solutions with $c > 1$ are only transiently accelerating, and $c \leq 1$ allows eternally accelerating expansion even for the k=0 cosmology. But $c < 1$ is not obtainable from classical compactifications! M-theory case is $c = 1.13$

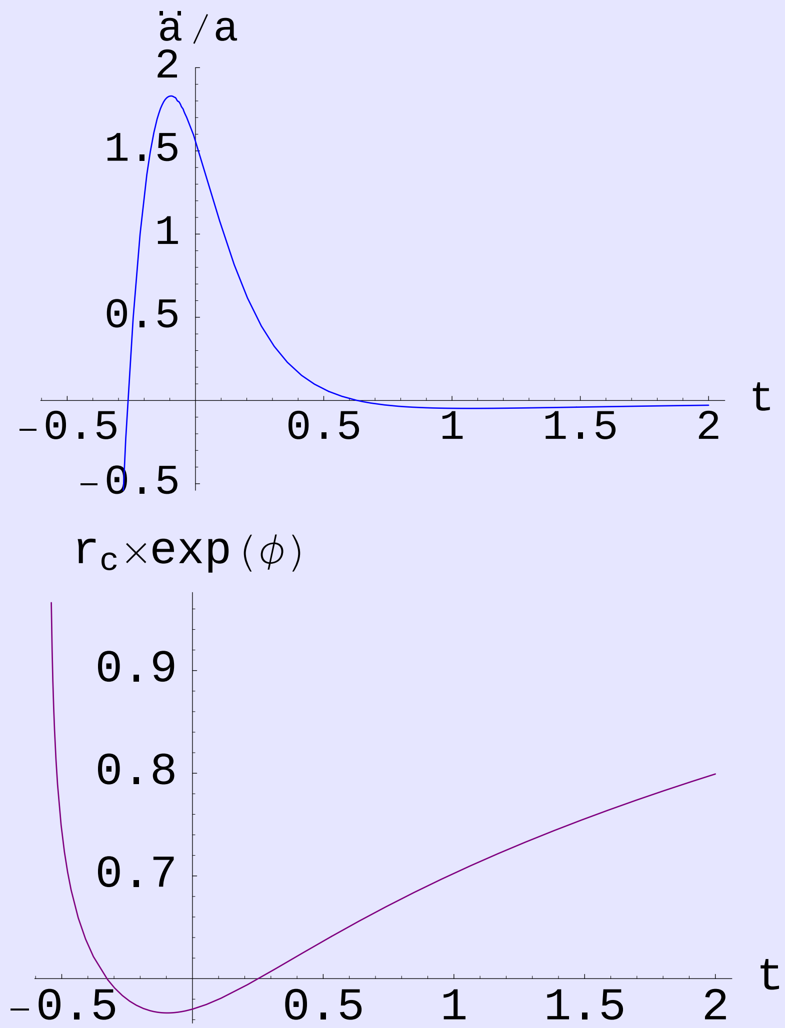
4d scale factor and Hubble parameter for k=0 cosmology





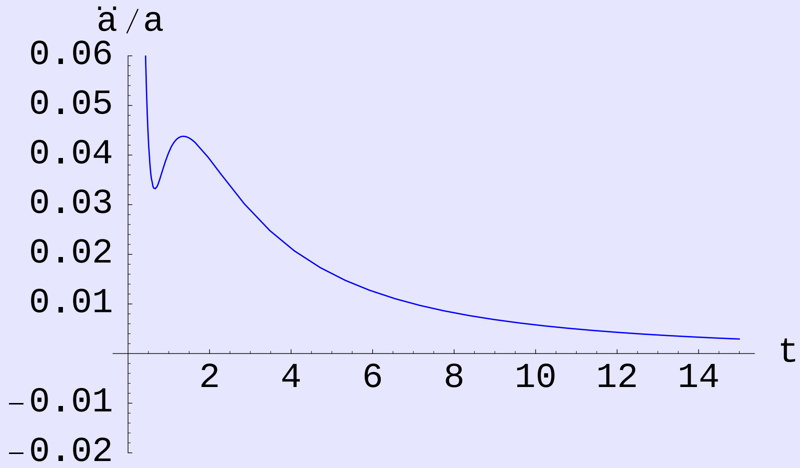
The parameters are fixed at $c = 3/\sqrt{7}$, $r_c = 0.4$, $\tilde{b} = 1$.

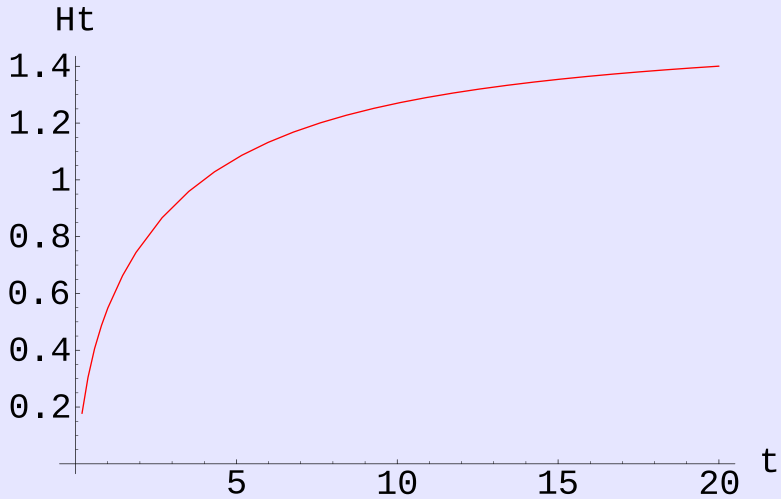
Transient acceleration and bouncing of internal volume



Parameter are fixed at $c = 3/\sqrt{7}$, $r_c = 0.4$, $\tilde{b} = 1$, $k = 0$, $k_1 = -1$.

An Example of Eternal Acceleration





The parameter values are $r_c = 0.8$, $\tilde{b} = 4$, $k = 0$, $k_1 = -1$, *and* $c = 0.8$

Open Universe and Eternally Accelerating Expansion

Next consider case of $k=-1$. In the zero-flux limit, the solution is

$$\mathbf{a}(\mathbf{t}) = \frac{\mathbf{c}}{\sqrt{\mathbf{c^2}-\mathbf{1}}}\,\mathbf{t} \equiv \mathbf{a_0}\,, \quad \varphi(\mathbf{t}) = \frac{\mathbf{1}}{\mathbf{c}} \ln\left(\frac{\mathbf{ct}}{\mathbf{r_c}}\right) \equiv \varphi_{\mathbf{0}}$$

This solution itself is not accelerating since $\ddot{a}(t)=0$, but, to the lowest order, where a non-zero field strength parameter $b>0$ serves as a source term, the solution is accelerating when $c<\sqrt{2}$.

To lowest order in cosmological perturbations

$$\mathbf{a}(\mathbf{t}) = \mathbf{a_0} + \mathbf{a_1}\,, \quad \varphi = \varphi_{\mathbf{0}} + \varphi_{\mathbf{1}}$$

the solution is

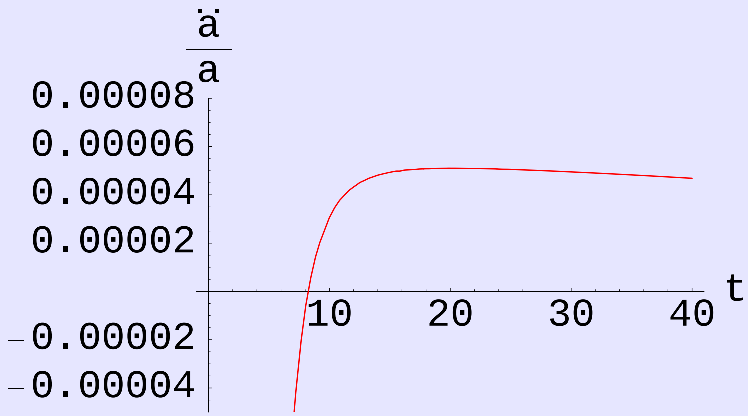
$$\mathbf{a_1} = \beta\,\mathbf{t^n}\,, \quad \varphi_{\mathbf{1}} = \beta\gamma\,\mathbf{t^{n-1}}$$

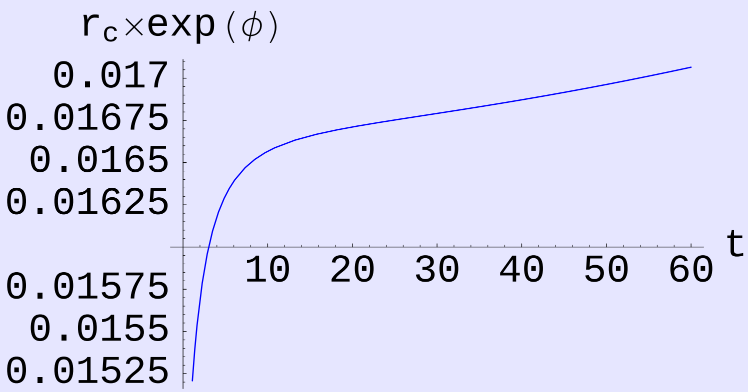
where β is undetermined (but a small constant) and

$$\gamma = \frac{\mathbf{3(1-n)}}{\mathbf{4}}\sqrt{\mathbf{c^2}-\mathbf{1}}\,, \quad \mathbf{n} = -\sqrt{\frac{\mathbf{4}-\mathbf{3c^2}}{\mathbf{c^2}}}$$

Full classical non-linear equations give an eternally accelerating solution even if $c>1$! This result holds with a non-trivial background fluxes

Eternally Accelerating Open Universe





We have set $r_c = 0.001$. For $r_c << 1$, $r_c e^{\phi}$ grows very slowly! See the next slide. In fact, for $k = 0$, the scale factor $a(t)$ grows much faster with t as compared to the $k = -1$ case. So $k = -1$ cosmology might tend towards spatial flatness in the subsequent evolution, in order to describe the universe we now observe

Internal space scale factors: Observationally too small

Table 1: The relation between r_c and the scale factors at $t = 10$

r_c	$a(t)$	$r_c\, e^{\phi}$	$\varphi - \varphi_0$
10^{-7}	5.4×10^{6}	0.8×10^{-5}	16.1
10^{-9}	1.8×10^{8}	2.2×10^{-7}	20.1
10^{-12}	4.0×10^{10}	1.0×10^{-9}	26.3

Table 2: The relation between r_c and the scale factors at $t = 1000$

r_c	$a(t)$	$r_c\, e^{\phi}$	$\varphi - \varphi_0$
10^{-7}	1.9×10^{8}	2.2×10^{-5}	20.5
10^{-9}	7.0×10^{9}	6.0×10^{-7}	25.5
10^{-12}	1.4×10^{12}	2.5×10^{-9}	30.5

Table 3: The relation between r_c and the scale factors at $t = 10^{10}$

r_c	$a(t)$	$r_c\, e^{\phi}$	$\varphi - \varphi_0$
0.1	1×10^{9}	35	22.5
0.001	4×10^{10}	1	26.5
0.00001	1.4×10^{12}	0.025	30.5

Summary

- A transient acceleration of the universe is generic for hyperbolic or flux compactifications of string/M-theory

- In the presence of matter fields, a short inflationary phase due to hyperbolic extra dimensions may be used to explain cosmic acceleration of the present universe.

- If a spatial curvature of the universe is flat and its accelerated expansion will continue forever then such an acceleration may arise from a scalar potential of the form $V \sim \exp(-2c\varphi)$ with the coupling constant $c \leq 1$

- However since for all known classical (string or Kaluza-Klein) compactifications on some non-trivial curved internals spaces and/or toroidal spaces with fluxes, only $c \gtrsim 1$ arises in practice we are led to explore alternatives and we find that an eternally accelerating expansion is possible for the coupling $c > 1$ only if the spatial curvature of the universe is negative.

- It seems possible to use M-theory motivated potential for dark energy.